Estimation complexity of chaotic oscillations in aspect of the shape of their trajectories

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The concept of the "*complexity*" of any object is its important of information-structural characteristic, and is one of the fundamental scientific concepts [1, 2]. Is no exception and a more narrow concept of the "*complexity of dynamic process*". With it linked the predictability and information capacity of processes. It is part of the many criteria for classifying the processes as deterministic, chaotic, stochastic. However, along with this, questions of definition and calculation the complexity of dynamic processes remains of methodologically open [2].

In this report, we proposed an original approach to the evaluation and analysis the complexity of chaotic sequences – through the study of their structural properties in the aspect of the forms of their trajectories in the terms of symbolic CTQ-analysis (the so-called TQ-complexity). The developed tools (based on Wolfram Mathematica) allows methods of computational physics us to study various phenomena in nonlinear multi-dimensional dynamical systems (including a network of oscillators with an arbitrary topology and configuration of the lattice).

Earlier, in the papers [3, 4], was presented computationally oriented method of symbolic CTQ-analysis, which allows a detailed study of the shape (structure of geometry) sequences $\{\mathbf{s}_k\}_{k=1}^{K}$ in the space $\mathrm{S}\times\mathrm{K}$, $\mathbf{s}\in\mathrm{S}\subset\mathbb{R}^N$, $k\in\mathrm{K}\subseteq\mathbb{N}$, $K\geq 3$. In coding a sequences $\{\mathbf{s}_k\}_{k=1}^{K}$ for each of its n th component ($n=\overline{1,\,N}$) is formed corresponding to a sequence of terms $\{T_k^{\alpha\varphi}\,|_n\}_{k=1}^{K}$ [3]. The transition $T_k^{\alpha\varphi}\,|_n\to T_{k+1}^{\alpha\varphi}\,|_n$ corresponds to the characters $Q^{\alpha\varphi}\,|_n$ that are are included in the alphabet $\mathrm{Q}_o^{\alpha\varphi}$.

In paper [3] put into consideration of the symbolic TQ-image of sequences $\{\mathbf{s}_k\}_{k=1}^{K}$, which formally defined an graph $\Gamma^{TQ}\,|_n=\left\langle \mathrm{V}^{\Gamma}\,|_n,\mathrm{E}^{\Gamma}\,|_n\right\rangle$, at wherein the vertex $\mathrm{V}^{\Gamma}\,|_n\subseteq\mathrm{T}_o^{\alpha\varphi}$, and the edges $\mathrm{E}^{\Gamma}\,|_n\subseteq\mathrm{Q}_o^{\alpha\varphi}$.

The complexity of chaotic trajectories in the framework of the approaches, possibly to evaluate on measures the complexity of of the graph $\Gamma^{TQ}\,|_n$: *degenerate* (DM) – by count of vertices and edges; *weighted* (WM) – subject to the specific value of complexity of $T^{\alpha\varphi}\,|_n$ and $Q^{\alpha\varphi}\,|_n$ symbols. Based on the fact that $\Gamma^{TQ}\,|_n$ – a weighted graph, each of the measures has two implementations: *topological* and *metric*. The metric is defined by the Boltzmann-Shannon entropy (for a DM) and Renyi (for a WM). By analyzing these measures, it is possible an study various aspects of the structure of chaotic attractors in nonlinear multi-dimensional dynamical systems.

In this paper, a numerical study the TQ-complexity of the trajectories of Rössler system:

$$\dot{x}=-y-z,\ \dot{y}=x+p\,y,\ \dot{z}=q+z(x-r),\ p=0.2,\ q=0.1,$$

for cases band-type $r=r_b=4.4$ and screw-type $r=r_s=12$ chaos. The results are compared with previously obtained data on the structure of these attractors [4].

References

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