Symbolic CTQ-analysis – a new method for studying of financial indicators

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Outline

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Key Idea – shape of trajectories

Denote the sequence (*such as the values of some financial indicators*):

$$\left\{ \{\mathbf{s}_k\}_{k=1}^{K},\ \{t_k\}_{k=1}^{K} \right\},$$

$$\mathbf{s} \in \mathrm{S} \subset \mathbb{R}^N, \quad t \in \mathrm{T} \subset \mathbb{R}, \quad t_{k+1} > t_k, \quad k \in \mathrm{K} \subset \mathbb{N},\ n = \overline{1,\ N},\ k = \overline{1,\ K}.$$

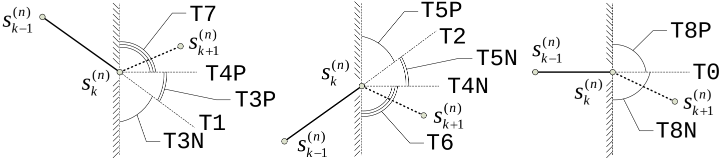
We introduce the map:

$$\{s_k^{(n)}\}_{k=0}^{K+1} \Rightarrow \{T_k^{\alpha\varphi}|_n\}_{k=1}^{K}, \quad T_k^{\alpha\varphi} = [T_k^{\alpha\varphi}|_1 \ldots T_k^{\alpha\varphi}|_N],$$

where $T^{\alpha\varphi}|_n$ – symbol of T-alphabet:

$$\mathrm{T}_o^{\alpha\varphi} = \{\mathtt{T0},\ \mathtt{T1},\ \mathtt{T2},\ \mathtt{T3N},\ \mathtt{T3P},\ \mathtt{T4N},\ \mathtt{T4P},\ \mathtt{T5N},\ \mathtt{T5P},\ \mathtt{T6},\ \mathtt{T7},\ \mathtt{T8N},\ \mathtt{T8P}\}.$$

We define symbol of the T-alphabet – select a subsequence $\{\mathbf{s}_{k-1},\ \mathbf{s}_k,\ \mathbf{s}_{k+1}\}$:

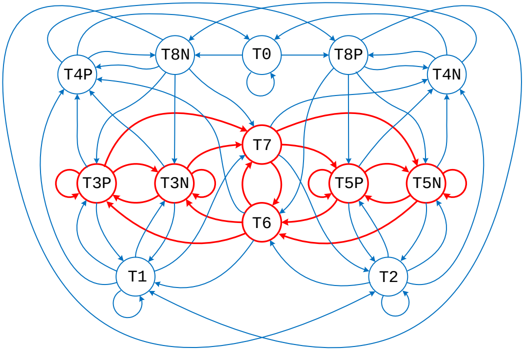


Symbolic TQ-image

Symbolic TQ-image of sequence $\{\mathbf{s}_k\}_{k=1}^{K}$

Directed graph $\Gamma^{\alpha\phi}|_n = \langle \mathrm{V}^{\Gamma}|_n, \mathrm{E}^{\Gamma}|_n \rangle$:

$\mathrm{V}^{\Gamma}|_n \subseteq \mathrm{T}_o^{\alpha\varphi}$ – vertex $\Gamma^{\alpha\phi}|_n$ and $\mathrm{E}^{\Gamma}|_n \subseteq \mathrm{Q}_o^{\alpha\varphi}$ – edges $\Gamma^{\alpha\phi}|_n$.



The main formalisms:

- CTQ-symmetry of trajectories;
- TQ-bifurcations;
- TQ-complexity;
- T-synchronization;
- Q-control.

Key article (in English)

- A.V.M., *Structure of Synchronized Chaos Studied by Symbolic Analysis in Velocity–Curvature Space, Technical Physics Letters, 38:2 (2012), 155—159, arXiv: 1203.4214.*
- A.V.M., *Multidimensional Dynamic Processes Studied by Symbolic Analysis in Velocity–Curvature Space, Computational Mathematics and Mathematical Physics, 52:7 (2012), 1017—1028.*
- A.V.M., *Measure of Synchronism of Multidimensional Chaotic Sequences Based on Their Symbolic Representation in a T-Alphabet, Technical Physics Letters, 38:9 (2012), 804–808, arXiv: 1212.2724.*

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The general idea of T-sync

Definition

The components of the sequence $\{\mathbf{s}_k\}_{k=1}^{K}$ are T-synchronized by a count k, if the corresponding sequence $\{T_k^{\alpha\varphi}\}_{k=1}^{K}$ following equality holds $J_{sym}^{\alpha\varphi}\left[T_k^{\alpha\varphi}\right] = 1$, where:

$$J_{sym}^{\alpha\varphi}\left[T_k^{\alpha\varphi}\right] = \begin{cases} 1 & T_k^{\alpha\varphi}|_1 = \ldots = T_k^{\alpha\varphi}|_n = \ldots = T_k^{\alpha\varphi}|_N, \\ 0 & \text{otherwise.} \end{cases}$$

Anti-synchronization: $s_k^{(n)} \to -1 \cdot s_k^{(n)}$.

+1	T0	T1	T2	T3N	T3P	T4N	T4P	T5N	T5P	T6	T7	T8N	T8P
−1	T0	T2	T1	T5P	T5N	T4P	T4N	T3P	T3N	T7	T6	T8N	T8P

Lag-synchronization:

$$\left\{T_k^{\alpha\varphi}|_1 \to T_{k+h_1}^{\alpha\varphi}|_1, \ \ldots, \ T_k^{\alpha\varphi}|_N \to T_{k+h_N}^{\alpha\varphi}|_N\right\}.$$

Integral factor and Time structure

Particular integral factor:

$$\delta^{\alpha\varphi}_{m,\mathbf{h}} = \frac{1}{K^* + 1 - k^*} \sum_{k=k^*}^{K^*} J\left[T^{\alpha\varphi}_k \,|\, \{m,\, \mathbf{h}\}\right],$$

where: $k^* = 1 + \max(h_1,\, \ldots,\, h_N)$, $K^* = K + \min(h_1,\, \ldots,\, h_N)$.

Full integral factor:

$$\delta^{\alpha\varphi} = \max_m \max_{\mathbf{h}} \delta^{\alpha\varphi}_{m,\mathbf{h}}, \quad 0 \leqslant \delta^{\alpha\varphi} \leqslant 1,$$

Definition

Synchronized domain SD – a collection of samples of the sequence $\{T^{\alpha\varphi}_k\}_{k=1}^{K}$, for which we have the condition:

$$SD_r : \left\{J^{\alpha\varphi}_{sym}\left[T^{\alpha\varphi}_{k'}\right] = 1, J^{\alpha\varphi}_{sym}\left[T^{\alpha\varphi}_{k^-}\right] = 0 \vee k^- = 0, \right.$$
$$\left. J^{\alpha\varphi}_{sym}\left[T^{\alpha\varphi}_{k^+}\right] = 0 \vee k^+ = K + 1\right\},$$

where $k' = \overline{b^{SD}_r,\, b^{SD}_r + L^{SD}_r}$, $k^- = b^{SD}_r - 1$, $k^+ = b^{SD}_r + L^{SD}_r + 1$, r – domain number, $r = \overline{1,\, R^{SD}}$, and besides $R^{SD} \leqslant (K+1) \operatorname{div} 2$.

Analytical characteristics of Time structure

Spectral density synchronous domains SD:

$$H^{SD}\left[L^{SD}\right] = \sum_{r=1}^{R^{SD}} \delta[L_r^{SD},\, L^{SD}],$$

Conditional entropy of the structure of synchronous domains, for $\delta^{\alpha\varphi} > 0$:

$$E_{cnd}^{SD} = -\sum_{i=1}^{K} P^{SD}\left[i\right] \ln P^{SD}\left[i\right], \quad P^{SD}\left[L^{SD}\right] = \frac{H^{SD}\left[L^{SD}\right]}{\sum\limits_{i=1}^{K} H^{SD}\left[i\right]}.$$

Relative conditional entropy structure of synchronous domains:

$$\Delta_E = \frac{E_{cnd}^{SD}}{\hat{E}_{cnd}^{SD}}, \quad \hat{E}_{cnd}^{SD} = \ln W, \quad W = \left\lfloor \frac{\sqrt{17 + 8\,\delta^{\alpha\varphi}\,K} - 3}{2} \right\rfloor.$$

Map of synchronization:

$$M_k^{SD} = \begin{cases} L_r^{SD} & b_r^{SD} \leqslant k \leqslant b_r^{SD} + L_r^{SD}, \\ 0 & \text{otherwise.} \end{cases}$$

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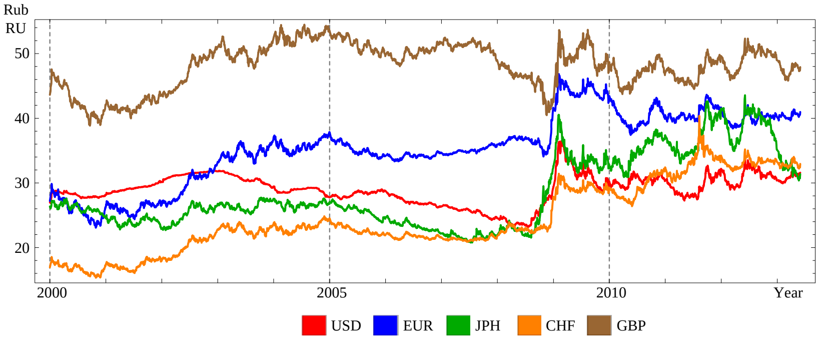
The rates of world currencies on Ruble



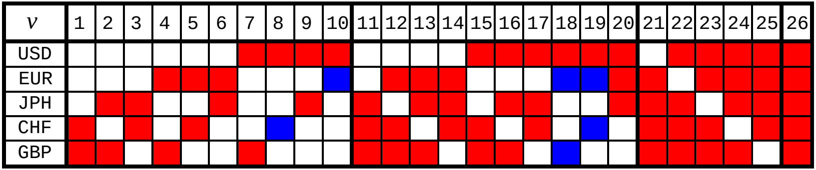
The period: from 01.01.1999 to 31.03.2013. The sample size: 3 545 counts.

The initial data are taken from the official web-site Central Bank of Russia: http://www.cbr.ru/eng/currency_base/dynamics.aspx

Remark: *Data extraction and processing was carried out in the program Wolfram Mathematica 9.*

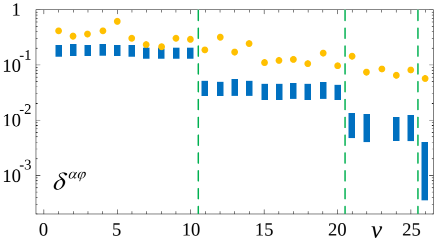
Integral factor of TQ-synchronization

The studied combinations:



– direct components   – anti- components: $s_k^{(n)} \to -1 \cdot s_k^{(n)}$

Integral factor of TQ-synchronization:

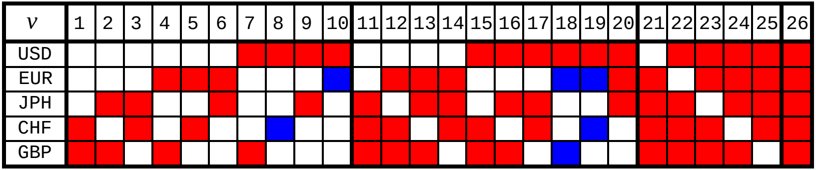


– real data [http://www.cbr.ru/eng/]

– synthetic data (homogeneous Markov chain, Monte Carlo simulation, 1000 trials, The ranges of values by in probability (empirical) 0.999)

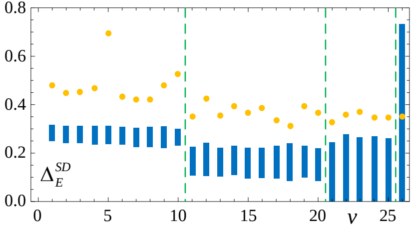
Time structure of TQ-synchronization

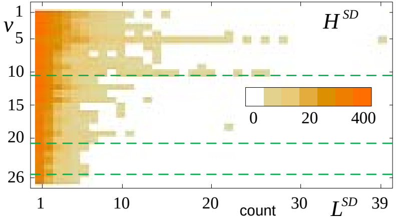
The studied combinations:



– direct components – anti- components: $s_k^{(n)} \to -1 \cdot s_k^{(n)}$

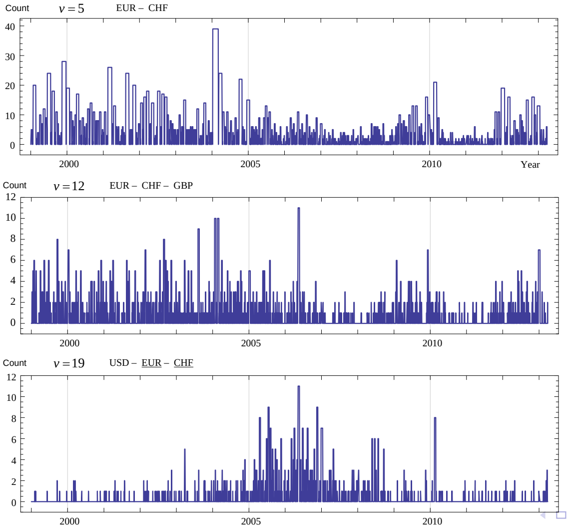
Entropy and Spectral density of synchronous domains:





Time structure of TQ-synchronization

Map of synchronization (samples for 3 combinations):



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Summary

- The CTQ-analysis method uses the term shape of the trajectory in the space $\mathrm{S} \times \mathrm{K}$.
- The strongest plus CTQ-analysis methods – focus on multidimensionality and nonstationarity studied processes and systems.
- For some currencies detected nonrandom long periods of simultaneous changes in their exchange rates.
- It is further planned multiscale CTQ-analysis of these financial indicators for investigation of their temporal structure in order to study the mechanism and causes of synchronicity.

Thank you for your attention!