The possibilities by symbolic analysis in velocity-curvature space: TQ-bifurcation, symmetry, synchronization

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Outline

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Symbolic Dynamics and Chaos

Nonlinear systems – ergodic (metric) approach

[A.Yu. Loskutov, Phys. Usp. 53, 1257 (2010)]

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Chaotic systems – "quasirandom"behavior

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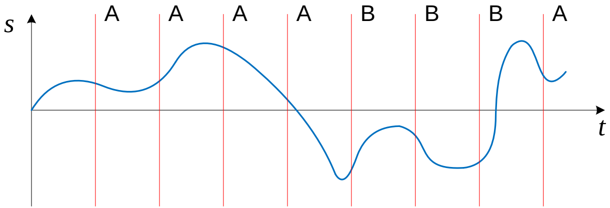
Classical Alphabets – computationally-oriented methods.

- Binarization of the space S
- Binarization of the space $\dot{\mathrm{S}}$
- M-cell decomposition of the space S
- Patterns of sequence $\{s_k\}$ – as symbols of the alphabet



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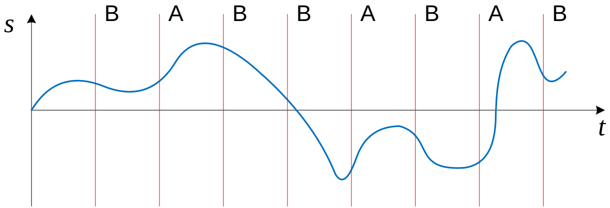

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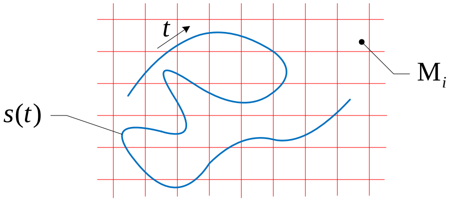

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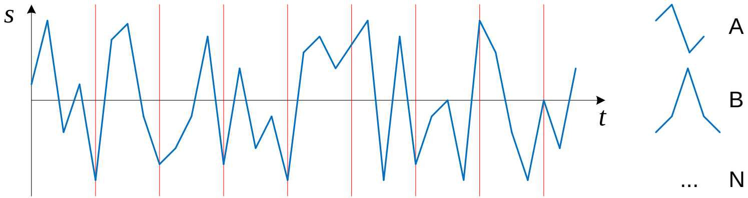


Possibilities of the classical alphabets

- The study of the structure of strange attractors;
- The construction of estimates of the invariant measure;
- The calculation of highest Lyapunov exponent;
- The analysis of hyperbolic systems;
- The structural stability of systems;
- The controllability of systems;
- Some of the applied aspects (neurophysiology, cardiology, finance, telecommunications).

The problems of classical alphabets

- Noninvariance to the transformations: $\mathbf{A}\,\mathbf{s} + \mathbf{b} \to \mathbf{s}$;
- Ambiguity of spaces partition: "one scheme – one system";
- Interpretation of borders the cell by the partition;
- The formalization of the criterion of minimum power for partition;
- Indirect connection with the topology of the trajectories.

Outline section

Key Idea – shape of trajectories

Denote the model of Dynamic System:

$$\mathbf{s}_{k+1} = \mathbf{f}\left(\mathbf{s}_k,\, \mathbf{p}\right), \quad \{\mathbf{s}_k\}_{k=1}^{K},$$

$$\mathbf{s} \in \mathrm{S} \subset \mathbb{R}^N, \quad \mathbf{p} \in \mathrm{P} \subset \mathbb{R}^L, \quad k \in \mathrm{K} \subset \mathbb{N},\, n = \overline{1,\, N},\, l = \overline{1,\, L},\, k = \overline{1,\, K}.$$

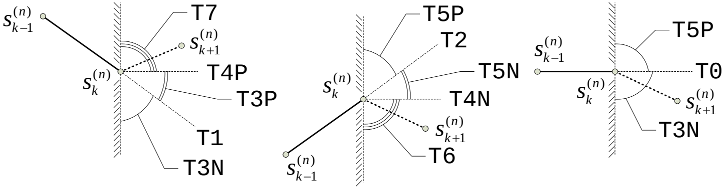
Hypothesis

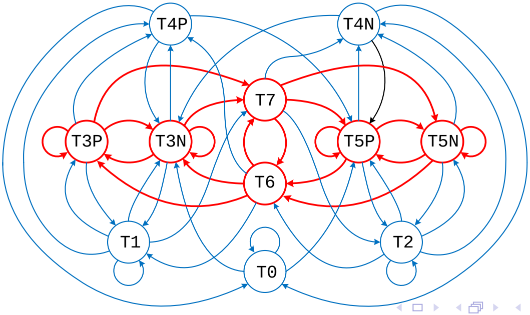
Shape of the trajectory sequence $\{\mathbf{s}_k\}_{k=1}^{K}$ in the space $\mathrm{S} \times \mathrm{K}$ adequately reflects (reveals) key internal properties of the nonlinear dynamical system, important things from the point of identification, controlled and prediction its evolution.

Definition

Characteristics shape the trajectory of the sequence $\{\mathbf{s}_k\}_{k=1}^{K}$ in the space $\mathrm{S} \times \mathrm{K}$ – it is certain its invariants, are preserved under uniform translation and dilation in the space S and homogeneous shifts in space K.

Scheme of the alphabet





The study of the dynamics of systems

Encoding shape the trajectories of dynamical systems – 11^N symbol.

TQ-symbolic image of the dynamic system (*)

Directed graph $\Gamma^{\alpha\phi} = \langle \mathrm{T}^{\alpha\varphi}, \mathrm{Q}^{\alpha\varphi} \rangle$:

$\mathrm{T}^{\alpha\varphi}$ – vertex $\Gamma^{\alpha\phi}$ – symbol of state,

$\mathrm{Q}^{\alpha\varphi}$ – edges $\Gamma^{\alpha\phi}$ – transitions between states.

Assen

TQ-symbolic image reflects the global structure dynamical system $\{\mathbf{f}^k, k \in \mathrm{K} \subset \mathbb{Z}\}$. There is a correspondence between the trajectories of the system in space $\mathrm{S} \times \mathrm{K}$, and the paths of graph $\Gamma^{\alpha\phi}$.

- CTQ-symmetry of trajectories;
- TQ-bifurcations;
- T-synchronization;
- Q-control.

* – echoes the approach of G.S Osipenko for M-cell partition of the space S of dynamic systems

Key article (in English)

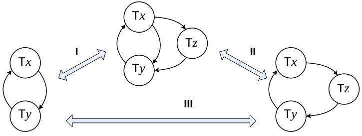
A.V.M., *Structure of Synchronized Chaos Studied by Symbolic Analysis in Velocity–Curvature Space, Technical Physics Letters, 38:2 (2012), 155—159, arXiv: 1203.4214.*

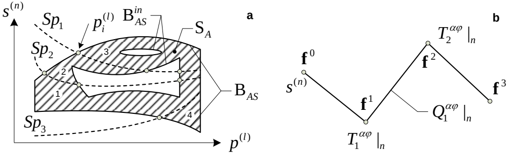
A.V.M., *Multidimensional Dynamic Processes Studied by Symbolic Analysis in Velocity–Curvature Space, Computational Mathematics and Mathematical Physics, 52:7 (2012), 1017—1028.*

A.V.M., *Measure of Synchronism of Multidimensional Chaotic Sequences Based on Their Symbolic Representation in a T-Alphabet, Technical Physics Letters, 38:9 (2012), 804–808* (in press).

Outline section

Maps of TQ-bifurcations





$$Sp^*_{k'}(\mathbf{p})|_n = \left\{\mathbf{s} : T^{\alpha\varphi}_{k'}(\mathbf{s},\, \mathbf{p})|_n = *\right\}, \quad * = \texttt{T0},\, \texttt{T1},\, \texttt{T2},\, \texttt{T4N},\, \texttt{T4P},$$

$$\mathbf{p}^*_{k'}|_n = \{\mathbf{p} : Sp^*_{k'}(\mathbf{p})|_n = \mathrm{B}_{AS}(\mathbf{p})\}, \quad \mathbf{p}^*_{k'}|_n = \left\{\mathbf{p} : Sp^*_{k'}(\mathbf{p})|_n = \mathrm{B}^{in}_{AS}(\mathbf{p})\right\}.$$

Sp^* – separatrix on symbol $*$
$\mathbf{p}^*$ – bifurcation point on symbol $*$
S_A – attractor of the mapping
B_{AS} – outer shell of the attractor
B^{in}_{AS} – inner shell of the attractor

$$\mathbf{s}_{k+1} = \mathbf{f}\,(\mathbf{s}_k,\, \mathbf{p})$$

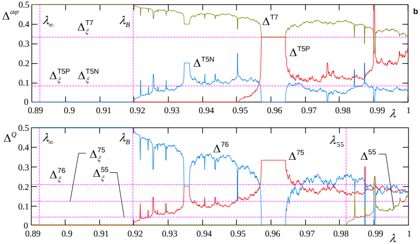
$$\mathbf{f} \in C^{\geqslant 2}(\mathbf{s}), \quad \mathbf{f} \in C^{\geqslant 0}(\mathbf{p})$$

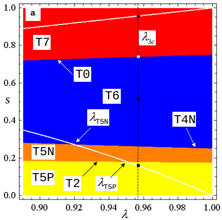
$$\mathbf{s}_k = \mathbf{f}^k(\mathbf{s_0},\, \mathbf{p}) = \mathbf{f}(\mathbf{f}(\ldots \mathbf{f}(\mathbf{s}_0,\, \mathbf{p}),\, \mathbf{p}),\, \mathbf{p})$$

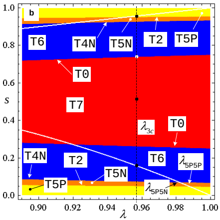
$$\mathbf{f}^0(\mathbf{s_0},\, \mathbf{p}) \equiv \mathbf{s_0}$$

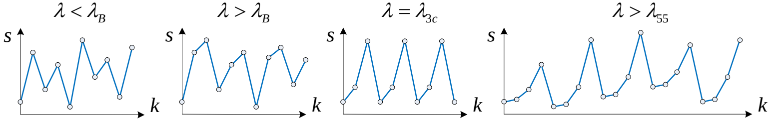
Sample: logistic mapping

$$s_{k+1} = 4\,\lambda\, s_k\,(1 - s_k), \quad s \in (0,\,1), \quad \lambda \in (0,\,1], \quad k \in \mathbb{Z}_{\geqslant 0}.$$









- Qualitatively different form of the trajectory
- New manifestation of the scale of the Feigenbaum

The general idea

Definition

The components of the sequence $\{\mathbf{s}_k\}_{k=1}^{K}$ are T-synchronized by a count k, if the corresponding sequence $\{T_k^{\alpha\varphi}\}_{k=1}^{K}$ following equality holds $J_{sym}^{\alpha\varphi}\left[T_k^{\alpha\varphi}\right] = 1$, where:

$$J_{sym}^{\alpha\varphi}\left[T_k^{\alpha\varphi}\right] = \begin{cases} 1 & T_k^{\alpha\varphi}|_1 = \ldots = T_k^{\alpha\varphi}|_n = \ldots = T_k^{\alpha\varphi}|_N, \\ 0 & \text{otherwise.} \end{cases}$$

Anti-synchronization: $s_k^{(n)} \to -1 \cdot s_k^{(n)}$.

+1	T0	T1	T2	T3N	T3P	T4N	T4P	T5N	T5P	T6	T7
−1	T0	T2	T1	T5P	T5N	T4P	T4N	T3P	T3N	T7	T6

Lag-synchronization:

$$\left\{T_k^{\alpha\varphi}|_1 \to T_{k+h_1}^{\alpha\varphi}|_1,\ \ldots,\ T_k^{\alpha\varphi}|_N \to T_{k+h_N}^{\alpha\varphi}|_N\right\}.$$

Integral factor and Time structure

Particular integral factor:

$$\delta^{\alpha\varphi}_{m,\mathbf{h}} = \frac{1}{K^* + 1 - k^*} \sum_{k=k^*}^{K^*} J\left[T^{\alpha\varphi}_k \,|\, \{m,\, \mathbf{h}\}\right],$$

where: $k^* = 1 + \max\left(h_1,\, \ldots,\, h_N\right)$, $K^* = K + \min\left(h_1,\, \ldots,\, h_N\right)$.

Full integral factor:

$$\delta^{\alpha\varphi} = \max_m \max_{\mathbf{h}} \delta^{\alpha\varphi}_{m,\mathbf{h}}, \quad 0 \leqslant \delta^{\alpha\varphi} \leqslant 1,$$

Definition

Synchronized domain SD – a collection of samples of the sequence $\{T^{\alpha\varphi}_k\}^K_{k=1}$, for which we have the condition:

$$SD_r : \left\{J^{\alpha\varphi}_{sym}\left[T^{\alpha\varphi}_{k'}\right] = 1, J^{\alpha\varphi}_{sym}\left[T^{\alpha\varphi}_{k^-}\right] = 0 \vee k^- = 0,\right.$$
$$\left. J^{\alpha\varphi}_{sym}\left[T^{\alpha\varphi}_{k^+}\right] = 0 \vee k^+ = K + 1\right\},$$

where $k' = \overline{b^{SD}_r,\, b^{SD}_r + L^{SD}_r}$, $k^- = b^{SD}_r - 1$, $k^+ = b^{SD}_r + L^{SD}_r + 1$, r – domain number, $r = \overline{1,\, R^{SD}}$, and besides $R^{SD} \leqslant (K+1)\,\mathrm{div}\,2$.

Analytical characteristics of Time structure

Spectral density synchronous domains SD:

$$H^{SD}\left[L^{SD}\right] = \sum_{r=1}^{R^{SD}} \delta[L_r^{SD},\, L^{SD}],$$

Conditional entropy of the structure of synchronous domains, for $\delta^{\alpha\varphi} > 0$:

$$E_{cnd}^{SD} = -\sum_{i=1}^{K} P^{SD}[i] \ln P^{SD}[i], \quad P^{SD}\left[L^{SD}\right] = \frac{H^{SD}\left[L^{SD}\right]}{\sum_{i=1}^{K} H^{SD}[i]}.$$

Relative conditional entropy structure of synchronous domains:

$$\Delta_E = \frac{E_{cnd}^{SD}}{\hat{E}_{cnd}^{SD}}, \quad \hat{E}_{cnd}^{SD} = \ln W, \quad W = \left\lfloor \frac{\sqrt{17 + 8\,\delta^{\alpha\varphi} K} - 3}{2} \right\rfloor.$$

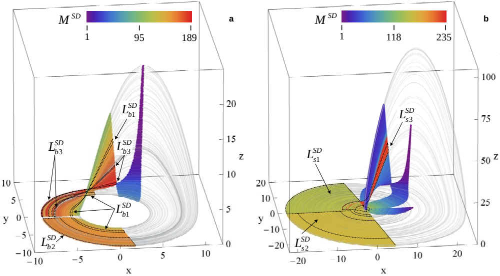
Map of synchronization:

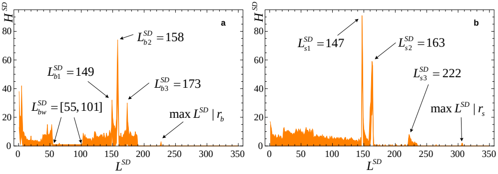
$$M_k^{SD} = \begin{cases} L_r^{SD} & b_r^{SD} \leqslant k \leqslant b_r^{SD} + L_r^{SD}, \\ 0 & \text{otherwise.} \end{cases}$$

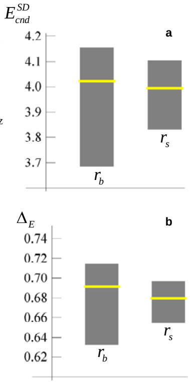
Sample: Rossler oscillator

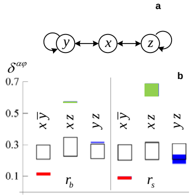
$$\dot{x} = -y - z, \quad \dot{y} = x + p\,y, \quad \dot{z} = q + z(x - r), \quad p = 0.2, \quad q = 0.1,$$

$$r = r_b = 4.4 \text{ – band-type chaos}, \quad r = r_s = 12 \text{ – screw-type chaos}.$$









(!) Chaos controlling

Outline section

Summary

- Developed a new approach to the symbolic analysis of multi-dimensional real sequences and discrete maps.
- CTQ-analysis method uses the term shape of the trajectory in the space $\mathrm{S} \times \mathrm{K}$.
- The strongest plus CTQ-analysis methods - focus on multidimensionality and nonstationarity studied processes and systems, including a sophisticated ensemble of non-identical oscillators large dimensions with arbitrary configuration and topology of the network (lattice).
- Introduced the concepts of:
  CTQ-symmetry, TQ-bifurcation, T-synchronization, T-control.
- Formalism CTQ-analysis has the potential to explore with a unified position of new types and mechanisms of synchronization of self-organization and control in nonlinear systems with chaotic dynamics.

Thank you for your attention!