Generalized synchronization of multidimensional chaotic systems in terms of symbolic CTQ-analysis

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Outline

Outline section

Generalized Synchronization and Symbolic Dynamics

- Generalized Synchronization of chaotic oscillations in form

$$\mathbf{y} = \mathrm{F}\left[\mathbf{x},\, \tau\right]$$

  is very important phenomena in physics (and not only in physics).
- But many important problems in this field remain unsolved: reliable detection, time structure, etc.
- In their turn Symbolic Dynamics is a very strongly substantiated tool for the analysis of nonlinear dynamical systems.
- It allows one to investigate complicated phenomena in systems such as chaos, strange attractors, hyperbolicity, structural stability, controllability, etc.

We have combined positions and obtain the new tool:

Generalized T-Synchronization.

Outline section

Definition of alphabet

Denote the discrete dynamical system:

$$\mathbf{s}_{k+1} = \mathbf{f}\left(\mathbf{s}_k,\, \mathbf{p}\right), \quad \{\mathbf{s}_k\}_{k=-\infty}^{\infty},$$

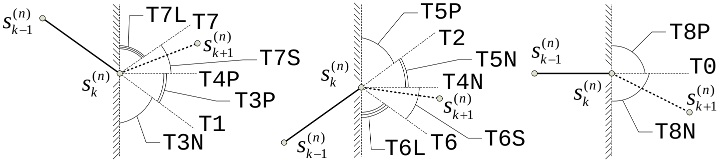
$$\mathbf{s} \in \mathrm{S} \subset \mathbb{R}^N, \quad k \in \mathrm{K} \subset \mathbb{N}, \quad \mathbf{p} \in \mathrm{P} \subset \mathbb{R}^M, \quad n = \overline{1,\, N},\, k = \overline{1,\, K},\, m = \overline{1,\, M}.$$

We introduce the primary mapping:

$$\left\{\mathbf{s}_{k-1}^{(n)},\, \mathbf{s}_k^{(n)},\, \mathbf{s}_{k+1}^{(n)}\right\} \Rightarrow T_k^{\alpha\varphi}|_n, \quad T_k^{\alpha\varphi} = \left[T_k^{\alpha\varphi}|_1 \ldots T_k^{\alpha\varphi}|_N\right], \quad \{T_k^{\alpha\varphi}\}_{k=1}^{K},$$

where $T^{\alpha\varphi}|_n$ – symbol of T-alphabet:

$$\mathrm{T}_o^{\alpha\varphi} = \{\mathtt{T0},\, \mathtt{T1},\, \mathtt{T2},\, \mathtt{T3N},\, \mathtt{T3P},\, \mathtt{T4N},\, \mathtt{T4P},\, \mathtt{T5N},\, \mathtt{T5P},\, \mathtt{T6S},\, \mathtt{T6},\, \mathtt{T6L},\, \mathtt{T7S},\, \mathtt{T7},\, \mathtt{T7L},\, \mathtt{T8N},\, \mathtt{T8P}\}.$$



Main Articles (in English)

A.V.M., *Structure of Synchronized Chaos Studied by Symbolic Analysis in Velocity–Curvature Space.* Technical Physics Letters, **38**:2 (2012), 155—159; arXiv: 1203.4214.

A.V.M., *Multidimensional Dynamic Processes Studied by Symbolic Analysis in Velocity–Curvature Space.* Computational Mathematics and Mathematical Physics, **52**:7 (2012), 1017—1028.

Outline section

The general idea of T-synchronization

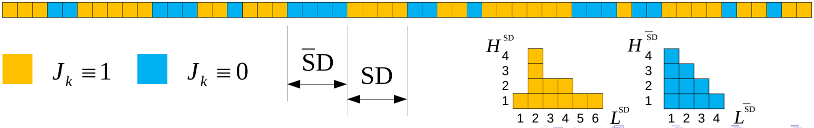
Remark

Let sequence $\{\mathbf{s}_k\}_{k=1}^{K}$ of dimension N is formed by the combination of the phase variables of N one-dimensional dynamical systems; i.e. $\mathbf{s}_k^{(n)}$ is the value of the phase variable of the nth system.

Definition

We will assume that the dynamical systems are synchronous at time instant kth in the sense of T-synchronization if the condition $J_k = 1$ is satisfied, where

$$J_k = \begin{cases} 1 & T_k^{\alpha\varphi}|_1 = \ldots = T_k^{\alpha\varphi}|_n = \ldots = T_k^{\alpha\varphi}|_N, \\ 0 & \text{otherwise.} \end{cases} .$$



Basic measures of T-synchronization

Anti-synchronization $\mathbf{s}_k^{(n)} \to -1 \cdot \mathbf{s}_k^{(n)}$ – inversion of symbols $T_k^{\alpha\varphi}|_n$:

$$\mathtt{T0} \leftrightarrow \mathtt{T0},$$

$$\mathtt{T1} \leftrightarrow \mathtt{T2}, \quad \mathtt{T3N} \leftrightarrow \mathtt{T5P}, \quad \mathtt{T3P} \leftrightarrow \mathtt{T5N}, \quad \mathtt{T4N} \leftrightarrow \mathtt{T4P},$$

$$\mathtt{T6S} \leftrightarrow \mathtt{T7S}, \quad \mathtt{T6} \leftrightarrow \mathtt{T7}, \quad \mathtt{T6L} \leftrightarrow \mathtt{T7L}, \quad \mathtt{T8N} \leftrightarrow \mathtt{T8P}.$$

Lag-synchronization – shift between components:

$$\left\{ T_k^{\alpha\varphi}|_1 \to T_{k+h_1}^{\alpha\varphi}|_1, \, \ldots, \, T_k^{\alpha\varphi}|_N \to T_{k+h_N}^{\alpha\varphi}|_N \right\}.$$

Partial integral coefficient of synchronism:

$$\delta^s_{m,\mathbf{h}} = \frac{1}{K^* + 1 - k^*} \sum_{k=k^*}^{K^*} J_k| \{m, \, \mathbf{h}\},$$

where $k^* = 1 + \max(h_1, \, \ldots, \, h_N)$ and $K^* = K + \min(h_1, \, \ldots, \, h_N)$.

Total integral coefficient of synchronism:

$$\delta^s = \max_m \max_{\mathbf{h}} \delta^s_{m,\mathbf{h}}, \quad 0 \leqslant \delta^s \leqslant 1.$$

The time structure of T-synchronization

We introduced the concept of a domains of two types:

synchronous domain SD

$$\mathrm{SD}_r : \left\{ J_{k'} = 1,\ J_{k''} = 0 \vee k'' = 0,\ J_{k'''} = 0 \vee k''' = K+1 \right\},$$

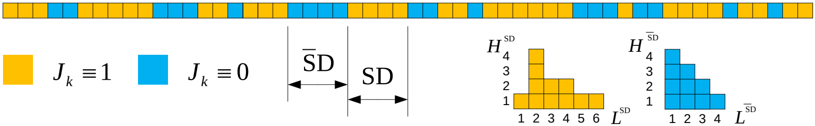
$$k' = \overline{b_r^{\mathrm{SD}},\, b_r^{\mathrm{SD}} + L_r}, \quad k'' = b_r^{\mathrm{SD}} - 1, \quad k''' = b_r^{\mathrm{SD}} + L_r^{\mathrm{SD}} + 1,$$

desynchronous domain $\overline{\mathrm{S}}\mathrm{D}$

$$\overline{\mathrm{S}}\mathrm{D}_r : \left\{ J_{k'} = 0,\ J_{k''} = 1 \vee k'' = 0,\ J_{k'''} = 1 \vee k''' = K+1 \right\},$$

$$k' = \overline{b_r^{\overline{\mathrm{S}}\mathrm{D}},\, b_r^{\overline{\mathrm{S}}\mathrm{D}} + L_r^{\overline{\mathrm{S}}\mathrm{D}}}, \quad k'' = b_r^{\overline{\mathrm{S}}\mathrm{D}} - 1, \quad k''' = b_r^{\overline{\mathrm{S}}\mathrm{D}} + L_r^{\overline{\mathrm{S}}\mathrm{D}} + 1,$$

$\vee$ is the symbol of the logical operation OR



The time structure of T-synchronization

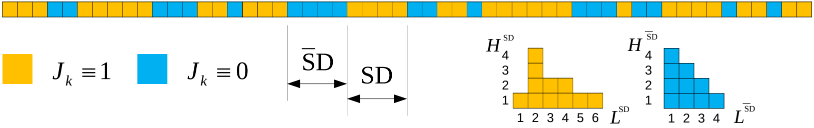
The spectral density function of synchronous domains SD:

$$H^{\mathrm{SD}}\,[L] = \sum_{r=1}^{R^{\mathrm{SD}}} \delta[L_r^{\mathrm{SD}},\, L],$$

where $\delta[\circ,\,\circ]$ is the Kronecker delta and $L = \overline{1,\,K}$.

The entropy of the structure of synchronous domains SD:

$$E^{\mathrm{SD}} = -\sum_{i=1}^{K} P^{\mathrm{SD}}\,[i]\,\ln P^{\mathrm{SD}}\,[i], \quad P^{\mathrm{SD}}\,[L] = \frac{H^{\mathrm{SD}}\,[L]}{\sum\limits_{i=1}^{K} H^{\mathrm{SD}}\,[i]}.$$



Main Articles (in English)

A.V.M., *Measure of Synchronism of Multidimensional Chaotic Sequences Based on Their Symbolic Representation in a T-Alphabet.* Technical Physics Letters, **38**:9 (2012), 804–808; arXiv: 1212.2724.

A.V.M., *Analysis of the Time Structure of Synchronization in Multidimensional Chaotic Systems.* J. Exp. Theor. Phys., **120**:5 (2015), 912–921; arXiv: 1505.04314.

The general idea of Generalized T-synchronization

Definition

We will assume that the dynamical systems are synchronous at time instant kth in the sense of generalized T-synchronization if the condition $J_k = 1$ is satisfied, where

$$J_k = \begin{cases} 1 & T_k^{\alpha\varphi} \in \mathrm{M}_{snc}^{FT}, \\ 0 & \text{otherwise.} \end{cases} .$$

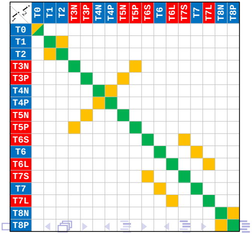
where M_{snc}^{FT} is set of symbols $T^{\alpha\varphi}$, which define synchronize state.

The symbols $T^{\alpha\varphi}$ are encoded in form of $\mathtt{T}\, i_1 \cdots \mathtt{T}\, i_n \cdots i_N$.

Basic requirements for the set M_{snc}^{FT}:

- $\left|\mathrm{M}_{snc}^{FT}\right| \leqslant \left|\mathrm{T}_o^{\alpha\varphi}\right|$,
- $\left|\mathrm{M}_{snc}^{FT}|_{i_n}\right| \leqslant 1,\ \forall \mathtt{T}\, i_n \in \mathrm{T}_o^{\alpha\varphi},\ n = \overline{1,\ N}$,

where $|\circ|$ is cardinality of set.



Construction of the set M_{snc}^{FT}

The objective function to fill the set M_{snc}^{FT}:

- Integral coefficient of synchronism: $\frac{1}{K}\sum_{i=1}^{K} i\, H^{SD}[i] \equiv \delta^s \to \max$,
- Length of synchronous domain: $\{\max L^{SD} \,:\, H^{SD}[L^{SD}] \geqslant 1\} \to \max$,
- ...

The number of variants is filling of the set M_{snc}^{FT}:

- Complete synchronization: $N_{snc}^{FT} = 1$,
- Antisynchronization: $N_{snc}^{FT} = 2^{N-1}$,
- Generalized synchronization:

$$N_{snc}^{FT} = \prod_{n=0}^{T-1} (T-n)^{N-1} = \left(\frac{2\,\mathtt{P}(3,\, T-1)}{T+1}\right)^{N-1},$$

where $\mathtt{P}$ is Pochhammer symbol, $T = \left|\mathrm{T}_o^{\alpha\varphi}\right|$.

Construction of the set M_{snc}^{FT}

The number of variants is filling of the set M_{snc}^{FT}.

Samples ($T = 17$ is standard T-alphabet):

- $N = 2, \quad N_{snc}^{FT} = 355\,687\,428\,096\,000,$
- $N = 3, \quad N_{snc}^{FT} = 126\,513\,546\,505\,547\,170\,185\,216\,000\,000.$
- $N \gg 2,$ Curse of dimensionality!

Who is to blame? What to do?

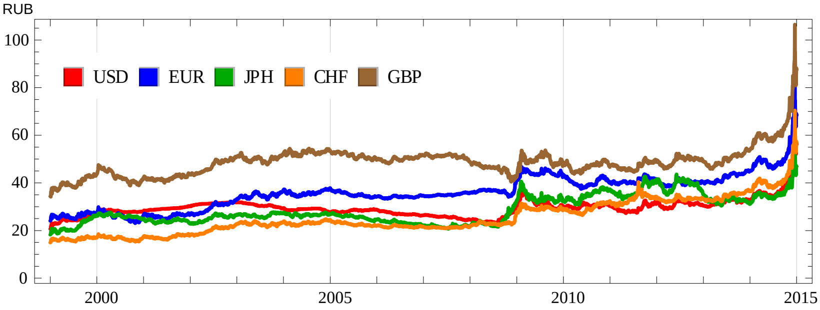
It is an open problem!

Variant: breadth-first search with a cut-off of bad branches.

Outline section

Exchange rates of some world currencies

The object of analysis is the time series of exchange rates of some world currencies (US dollar [USD], Euro [EUR], Japanese Yen [JPH], Swiss Franc [CHF], and British Pound [GBP] against Russian ruble).



The analyzed period is from 01.01.1999 to 31.12.2014.

The original data are taken from the official web-site of the Central Bank of Russia (Bank of Russia, exchange rates, www.cbr.ru/eng/).

Synchronization of USD and EUR

Short sample: USD and EUR.

- Complete synchronization: $\delta^s = 0.174492$,
- Antisynchronization: $\delta^s = 0.219433$,
- Generalized synchronization: $\delta^s = 0.222948$.

	T0	T1	T2	T3N	T3P	T4N	T4P	T5N	T5P	T6S	T6	T6L	T7S	T7	T7L	T8N	T8P
T0	0	0	0	5	6	0	0	3	6	8	0	5	11	1	5	0	1
T1	0	0	0	1	4	0	0	1	4	3	0	3	4	0	2	0	0
T2	0	0	0	0	0	0	0	0	1	2	0	0	2	0	0	0	0
T3N	0	0	0	96	54	0	0	42	110	35	0	55	29	2	49	1	0
T3P	0	0	1	56	108	2	1	124	51	66	2	37	53	1	43	0	0
T4N	0	0	0	7	8	1	1	8	8	5	0	6	10	1	4	1	0
T4P	2	0	0	8	6	1	1	7	9	16	0	8	10	0	7	2	0
T5N	0	0	0	34	112	0	2	93	49	50	0	20	60	0	31	0	1
T5P	0	0	0	93	36	1	0	50	117	26	0	49	37	1	68	1	0
T6S	0	0	0	20	41	2	0	45	44	89	2	35	107	0	34	1	1
T6	0	0	0	2	0	0	0	2	0	0	0	2	2	1	2	0	0
T6L	0	0	0	49	28	0	1	27	56	36	0	59	47	0	109	0	1
T7S	0	0	0	35	60	0	0	38	33	110	0	44	69	0	29	0	0
T7	0	0	0	0	3	1	0	1	1	1	0	1	0	0	0	0	1
T7L	0	0	0	72	30	0	0	33	37	37	0	104	35	1	59	0	1
T8N	0	1	0	8	11	1	0	12	4	7	0	14	4	0	5	2	0
T8P	0	0	0	7	4	0	2	4	11	6	0	11	9	1	11	2	1

Short conclusions:

- The structure of synchronicity USD and EUR is more complex than the Complete or Anti-.
- Generalized synch is a combination of Anti- and Complete- synch.

Outline section

Summary

- In this report, we proposed an original approach to the evaluation and analysis generalized synchronization of chaotic sequences.
- The real experiment demonstrated the efficiency measures of generalized T-synchronization.
- The developed tools expand methods of computational physics for study various phenomena in nonlinear multi-dimensional dynamical systems.
- At the moment, we are resolving one open problem:
  - The effective algorithms for filling set M_{snc}^{FT}.

Thank you for your attention!